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Abhandlung

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From Seeberg to Colmar: early mathematical concepts in prehistoric Europe at the interface between material culture, technology and metaphors

Abstract: Im Mittelpunkt dieses Artikels stehen Vergleichsanalysen von Kupferperlen der Cortaillod-Kultur sowie einleitend das Depot von Seeberg Burgäschisee-Süd. Dieses Depot war für eine lange Zeit ein isolierter Fundkomplex, sodass vonseiten der Forschung Thesen zu dessen metrologischer Struktur als wenig überzeugend angesehen wurden. 2008 jedoch kam es zu einem Durchbruch für die Forschung, konnte doch in Colmar in einem äneolithischen Grab eine für die Cortaillod-Kultur charakteristische Perlenkette geborgen werden, deren Platzierung in der Bestattung eine Bewertung der Stellung und Bedeutung jener Perlen während des Äneolithikums erlaubte. Für die hier vorgelegten Analysen sind diese Perlen von großer Bedeutung, vergleichbar etwa mit dem „Stein von Rosette" für die Entzifferung der Hieroglyphen, tragen sie doch dazu bei, die archaischen und wenig abstrakten Methoden mathematischen Denkens und verwendeter Zahlbegriffe zu verstehen und beides mit dem damals neuen Ansatz der Metallurgie zu verknüpfen.

Keywords: Kupferperlen; Cortaillod-Kultur; Zahlenbegriff; Metallurgie, Metapher

Résumé: L'article ci-dessous est une étude comparative de perles de cuivre appartenant à la culture de Cortaillod. Le célèbre dépôt de Seeberg Burgäschisee-Süd est resté unique pendant de longues années, ce qui amena certains archéologues à douter de la validité d'une hypothèse d'une structure métrologique. Suivant un aperçu sur cette trouvaille, notre article présente une analyse basée sur la méthode statistique de Kendall. De plus, nous disposons maintenant d'une nouvelle découverte de la plus haute importance : un autre dépôt de perles, mis à jour lors de fouilles à Colmar en Alsace. Ces perles, d'un type caractéristique de la culture de Cortaillod, furent retrouvées dans une tombe énéolithique. La répartition de ces objets dans la sépulture lève enfin le doute sur la valeur que l'on plaçait sur ces perles. L'étude comparative de ces deux dépôts, qui fait appel à des méthodes tant statistiques que conventionnelles, constitue le noyau de cet article ; elle nous permet d'approfondir nos connaissances dans le domaine problématique des notions mathématiques et métrologiques existant en Europe à l'époque préhistorique. Les perles de la culture de Cortaillod se trouvent à l'intersection de méthodes plus archaïques et moins abstraites dans le domaine des mathématiques et du dénombrement et des nouvelles notions liées à l'apparition de la métallurgie.

Mots-clefs: perles de cuivre; culture de Cortaillod; notion de nombre; métrologie; métaphore

Abstract: This article presents a comparative analysis of copper beads of the Cortaillod culture. The famous deposit from the site of Seeberg Burgäschisee-Süd remained isolated for a long time, and hence the hypothesis of its metrological structure seemed unbelievable to some archaeologists. This article begins with a reminder of this find and presents a new analysis based on Kendall's statistics. In addition, we present a breakthrough discovery of another deposit of beads. During excavations of the site of Colmar in Alsace, an Eneolithic grave was discovered; it produced copper beads of a type that is characteristic of the Cortaillod culture. The distribution of these objects in a grave finally removes doubts as to how beads were valued. The comparison of the deposits of Seeberg and Colmar, using both statistical and conventional methods, is at the core of this article; it allows us to gain a deeper insight into the problematic area of early mathematical and metrological concepts in prehistoric Europe. The beads lie at

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Fig. 1: Two strings of copper beads found at Seeberg, Burgäschisee-Süd (after Strahm 1994)

the interface between traditional, more archaic and less abstract methods of perceiving mathematical relations based on the concept of linear measure and the new approach brought about by the arrival of metal technology.

Keywords: copper beads; Cortaillod culture; number concept; metrology, metaphor

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Introduction

I would like to begin with a short summary of the state of research. The original discovery consists of two strings of copper beads recovered from a shallow pit close to one of the dwelling structures excavated in 1967 at the Late Neolithic Cortaillod culture settlement of Seeberg, Burgäschisee-Süd[1] on the Swiss plateau. There were no traces of a pouch or of any kind of container accompanying the beads. The leather strings had small knots at both ends preventing the beads from slipping off. All the beads were well-preserved; the fact that they displayed various degrees of oxidation suggests that they had not always been together as two strings[2]. It was an interesting and provoking discovery indeed. Beads of this type are usually found singly in settlements of the Cortaillod culture in Switzerland. We should thus take a closer look at the Seeberg find[3].

The two strings, designated K1 (with 18 beads) and K2 (with 36 beads), comprise 54 beads in total (Fig. 1). Two of them have metal fillings and all of them carry traces of wear, although their interpretation as ornaments was rejected by their discoverers[4]. Ottaway and Strahm 1975[5] also stressed two further aspects. Firstly, the numbers of beads on the two strings represent a straightforward mathematical proportion (36 + 18). Secondly, the two strings clearly differ in terms of bead weights, there being twice as many light beads as there are heavy beads (Fig. 2). The 36 beads of string K2 each weigh between 0.6 to 8.8 g, while the 18 beads of the shorter string K1 each weigh between 8.1 and 17.3 g.

Ottaway and Strahm went on to say that the different weights and shapes of the beads rule out the possibility that they were objects of a standardised value. In an earlier publication, Sangmeister and Strahm[6] distinguished six types of beads. Typological and metallographic analyses of the beads discussed here would suggest that the beads came from different metal workshops (different types and chemical compositions) but from a single source. Also worth mentioning here is that the aforementioned analy-

1 Sangmeister/Strahm 1974.
2 Ottaway/Strahm 1975, 311.
3 See Dzbyński 2008a.
4 Sangmeister/Strahm 1974.
5 Ottaway/Strahm 1975.
6 Strahm 1974, 255.

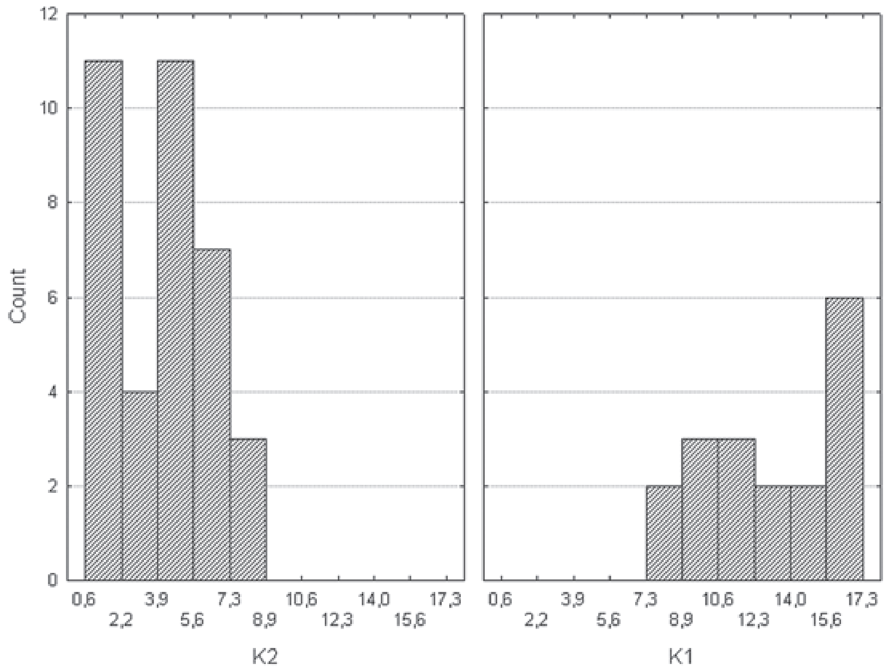

Fig. 2: Distribution of the weight variables of beads recovered from Seeberg, Burgäschisee-Süd. K2: longer string of 36 beads; K1: shorter string of 18 beads

ses show the Seeberg finds to be very much like those of the Mondsee culture[7].

Some very important insights, that allow us to consider the find from a new perspective, have now been gained. Looking closely at the manufacturing method of the beads it was found that they were made from three or four metal rods. These were divided into a certain amount of beads and folded, which made it easier to transport. Thus, as Sangmeister and Strahm wrote[8], we are dealing with an early form of a convenient exchange medium. Both the beads and their output shape were a form of ingots, which were then manipulated in a rather complicated way, as we shall see below. Several years later Ottaway and Strahm[9] proposed to treat the beads from Seeberg as a specific medium of exchange – 'special purpose money'. The most important question, however, remained unanswered: since we are dealing with a form of currency, how would it be used, valued and counted?

Analyses

It turns out that the lack of standardisation of the beads from Seeberg, Burgäschisee-Süd is only apparent. A more detailed analysis revealed a hitherto overlooked characteristic of this deposit, namely that the bead weight distributions for both strings are bimodal, which is to say that two weight categories can be distinguished for each of the strings, i.e. 0.7–3.0 g and 3.0–7.0 g for string K2, and 8.0–14.0 g and 14.0–17.0 g for string K1. Not all of these weight categories can be distinguished with equal precision, this being probably due in part to post-depositional factors and the already mentioned differences in the degree of wear (which in turn probably was the result of the different 'biographies' of the various beads). Be that as it may, the weight categories that were distinguished can be discerned even today without any serious difficulty[10].

We shall now turn to defining the weight categories discernible in Fig. 2, based on the weight figures shown in Tab. 1. Although we are dealing with approximate figures, it is clear that the entire range of variability has a common denominator of *c.* 5.5 g (Group 2). We have groups of beads which double and triple that value (Groups 3 and 4 respectively). The least distinct Group 1 should represent half the common denominator and the fact that it fails to fit the

7 Strahm 1994, 20.
8 Sangmeister/Strahm 1974.
9 Ottaway/Strahm 1975.
10 See Dzbyński 2008a.

Tab. 1: Kendall analysis of the copper beads from Seeberg, Burgäschisee-Süd (see also Fig. 3)

	N	Mean	Min	Max	St.dev.
Group 1	14	**1,55**	0,57	2,54	0,65
Group 2	22	**5,69**	3,49	8,05	1,23
Group 3	12	**9,74**	7,28	12,80	1,74
Group 4	8	**16,05**	14,74	17,32	0,90

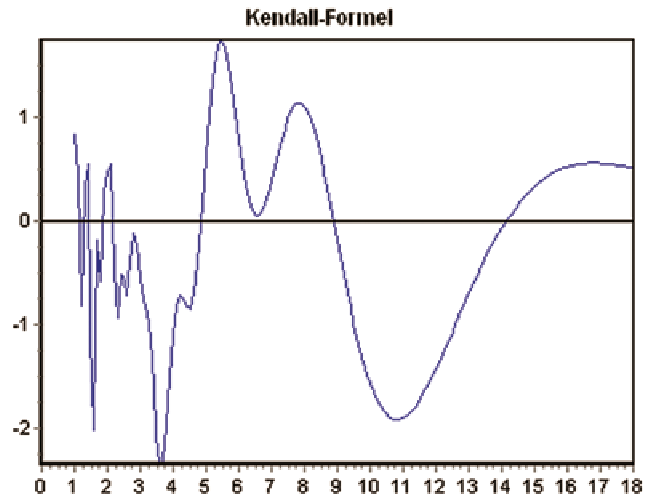

Fig. 3: Kendall analysis of the copper beads from Seeberg, Burgäschisee-Süd (see also Tab. 2)

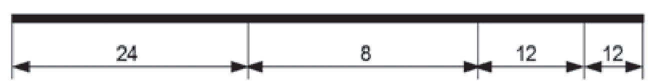

Fig. 4: The copper beads from Seeberg, Burgäschisee-Süd could be produced from one single rod of copper. The numbers represent the quantity of beads made from specific fragments of the rod

suggested pattern may be owed to various factors, most probably to wear or post-depositional processes which were most severe in the case of the smallest beads.

What we therefore see in the Seeberg beads is a simple metrological structure involving manipulations of the basic value of *c.* 5.5 g. This seems to be the most logical interpretation for the following reasons. Metrological systems in antiquity were based on simple rules of proportionality[11] whereby specific units were either halved or doubled. Assuming in our case that the basic unit was the maximum value of the weight variable (*c.* 16 g in Group 4; see Tab. 1), then half that value would fall precisely midway between the maxima represented by Groups 2 and 3. This would obviously be out of line with the observed regularity. In the case of the Seeberg beads the figure that best fits the regularity is 5.5 g, which, incidentally, also happens to be represented by the largest number of beads.

We must not be discouraged by the fact that by today's standards these are not precisely defined weight categories. Given the rules of ancient metrology, measurement units may be identified only according to their distribution which should be close to normal for the individual variable peaks[12]. We know why some of the distributions of ancient variables differ from the classical bell curves[13]; post-depositional processed may be invoked as some of the reasons for this. We also have mathematical models which provide good descriptions of these phenomena[14].

Although the weight of the beads is a clear indication that a mathematical mind is hidden in their realisation, I do not think that the weight need be the most appropriate factor that defines the beads. This issue has already been alluded to above. But before attempting a final interpretation let me go through some statistical tests.

In my early studies I used the Broadbent method to discover metrological structures. This method is based on taking account of the normal distribution, and test whether individual variable sub-distributions, suspected of bearing a metrological structure, are located at equal distances from each other. In short, it consists of observing histograms and using simple mathematical formulae. The weakness, however, was the need to have quite numerous variables. The conclusions reached through the Broadbent method can now be confirmed by using a different statistical method designed to test the metrological structures. Kendall's analysis[15] offers such a possibility, by which the author himself refuted the thesis of a megalithic yard stated by Thom[16].

Kendall's analysis does not have the limitations of the method that Broadbent used, although it set itself other goals. Instead of a comprehensive follow-up distribution, it can only predict the most likely value, which participates in the rule of doubling in the whole metrological structure. This value is then defined as the unit of measure that is sought. Kendall's method is commonly used by archaeologists involved in the study of early metrological systems[17]. In our case, the analysis indicates the amount of 5.5 g as best reflecting the rule of the whole set (Fig. 3, Tab. 2), al-

11 Petruso 1992, 10.
12 Broadbent 1955, 46.
13 Petruso 1992, 70.
14 Broadbent 1955; Dzbyński 2004.
15 Kendall 1974.
16 Thom 1967.
17 Pare 1999; Rahmstorf 2010.

Tab. 2: Weight groups of beads distinguished on the basis of the weight distributions illustrated in Fig. 2

— -**** — -Kendall-Formel — -**** — -

1,0	0,8364	3,9	-1,5187	6,8	0,1632	9,7	-1,2471	12,6	-0,9895	15,5	0,4523
1,1	0,4441	4,0	-1,0915	6,9	0,2669	9,8	-1,3654	12,7	-0,9152	15,6	0,4699
1,2	-0,8082	4,1	-0,8149	7,0	0,3889	9,9	-1,4726	12,8	-0,8414	15,7	0,4858
1,3	0,3367	4,2	-0,7162	7,1	0,5204	10,0	-1,5682	12,9	-0,7683	15,8	0,4999
1,4	0,5530	4,3	-0,7478	7,2	0,6530	10,1	-1,6519	13,0	-0,6962	15,9	0,5123
1,5	-1,0961	4,4	-0,8225	7,3	0,7791	10,2	-1,7238	13,1	-0,6253	16,0	0,5231
1,6	-2,0175	4,5	-0,8521	7,4	0,8926	10,3	-1,7839	13,2	-0,5559	16,1	0,5324
1,7	-0,1876	4,6	-0,7741	7,5	0,9881	10,4	-1,8324	13,3	-0,4880	16,2	0,5403
1,8	-0,5822	4,7	-0,5645	7,6	1,0619	10,5	-1,8696	13,4	-0,4219	16,3	0,5467
1,9	0,2657	4,8	-0,2355	7,7	1,1112	10,6	-1,8958	13,5	-0,3577	16,4	0,5518
2,0	0,4683	4,9	0,1738	7,8	1,1345	10,7	-1,9115	13,6	-0,2955	16,5	0,5557
2,1	0,5450	5,0	0,6109	7,9	1,1314	10,8	-1,9172	13,7	-0,2354	16,6	0,5584
2,2	-0,4741	5,1	1,0227	8,0	1,1022	10,9	-1,9135	13,8	-0,1775	16,7	0,5600
2,3	-0,9294	5,2	1,3647	8,1	1,0480	11,0	-1,9010	13,9	-0,1218	16,8	0,5605
2,4	-0,5084	**5,3**	**1,6068**	8,2	0,9705	11,1	-1,8802	14,0	-0,0685	16,9	0,5601
2,5	-0,5642	**5,4**	**1,7350**	8,3	0,8720	11,2	-1,8518	14,1	-0,0176	17,0	0,5587
2,6	-0,7141	**5,5**	**1,7499**	8,4	0,7550	11,3	-1,8164	14,2	0,0310	17,1	0,5565
2,7	-0,4042	**5,6**	**1,6646**	8,5	0,6222	11,4	-1,7746	14,3	0,0772	17,2	0,5535
2,8	-0,1117	**5,7**	**1,5001**	8,6	0,4765	11,5	-1,7272	14,4	0,1210	17,3	0,5498
2,9	-0,2379	5,8	1,2818	8,7	0,3207	11,6	-1,6747	14,5	0,1624	17,4	0,5454
3,0	-0,5542	5,9	1,0360	8,8	0,1578	11,7	-1,6176	14,6	0,2014	17,5	0,5404
3,1	-0,7557	6,0	0,7871	8,9	-0,0096	11,8	-1,5567	14,7	0,2381	17,6	0,5349
3,2	-0,8787	6,1	0,5559	9,0	-0,1790	11,9	-1,4923	14,8	0,2725	17,7	0,5288
3,3	-1,1312	6,2	0,3583	9,1	-0,3479	12,0	-1,4252	14,9	0,3046	17,8	0,5223
3,4	-1,5766	6,3	0,2051	9,2	-0,5141	12,1	-1,3557	15,0	0,3345	17,9	0,5153
3,5	-2,0577	6,4	0,1019	9,3	-0,6758	12,2	-1,2844	15,1	0,3622	18,0	0,5081
3,6	-2,3443	6,5	0,0499	9,4	-0,8313	12,3	-1,2117	15,2	0,3877		
3,7	-2,3070	6,6	0,0468	9,5	-0,9791	12,4	-1,1381	15,3	0,4112		
3,8	-1,9812	6,7	0,0870	9,6	-1,1180	12,5	-1,0639	15,4	0,4327		

though, as we recall, the size should be still regarded with some caution[18].

Verification: the beads from Colmar

The deposit from Seeberg remained an isolated find for a long time, leading some archaeologists to doubt the hypothesis of its metrological structure. It should be noted, however that beads of this type are a fairly common feature of the Cortaillod culture in Switzerland. Unfortunately, most of these beads were found without an explicit context, which makes a comprehensive analysis harder. But in 2008 a breakthrough discovery was made[19]. During rescue excavations in Colmar, a site in Alsace, an Eneolithic grave was discovered, furnished with copper beads of a type characteristic of the Cortaillod culture. The distribution of these objects in the grave finally removes doubts as to how the beads were valued, as we shall see below.

Three graves were uncovered in the excavations, containing four individuals. The most interesting burial is a grave containing the skeleton of an adult lying in an unusual position. The grave was equipped with just one vessel and three clusters of beads that were placed in a very characteristic way. A necklace containing 25 beads was found near the foot of the deceased. The second necklace was found on his waist, while the third group of four beads was discovered under the skeleton, at chest level. The placement of these three groups suggests that the beads were fastened to strings, like those of Seeberg, and attached to the body in some way. The excavators ex-

18 The application for computing the Kendall-formula was developed by Matthias Zimmer, a former student of the University of Mainz (today a PhD holder).

19 Lefranc *et al.* 2009.

pressed the opinion that this burial, both in its unusual arrangement which may have been manipulated and by the deposition of 400 g of copper in the form of 56 beads, must be seen as exceptional[20].

The details of the beads found by the deceased are illustrated in Fig. 5[21]. The diagrams show histograms of their weight in three groups. Group 1 contains the lightest pieces, which encircled the body of the deceased. Their weight is low and varies between 1 and 4 g. The light weight also explains the small spread of the variable where the beads seem to have an equal weight. Another group (2) is much more diffuse and contains the whole spectrum of the variable. This group was placed by the feet of the deceased. The third group (3) from the chest contains four items ranging from 6 to more than 12g.

Comparing these groups with the histogram of the whole variable at the bottom of the diagram, we can easily discover the following. The deceased was girdled with beads of the first type-weight group (1). On the chest there were four beads of the second type-weight group (3), while the third group of beads, containing the whole spectrum of the variable, was placed by his feet (2).

The observation that the weight distribution of the Colmar beads is not the same as those of Seeberg is also of importance. Does it not disprove the entire hypothesis? No, not really. In both cases a very similar amount of material was used (382 g at Seeberg and 400 g at Colmar) but at Colmar the weight spectrum of the beads is almost twice as large, as shown by the three heaviest pieces weighing about 30 g. The heaviest items from Seeberg weighed only about 16 g. The heaviest Colmar exemplars are therefore twice as heavy as those from Seeberg. This means that in the case of Colmar the copper rods have been manipulated differently (exponential in statistic sense) to prepare the beads. However, four metrological groups can be observed in both deposits.

Discussion

Mathematical and metrological studies rarely go further back than the Bronze Age for two reasons. First, researchers doubt that their search for mathematics so deep in prehistory makes sense. Besides the megalithic yard theory, which is controversial and in my opinion misunderstood, there is no research that is heading in this direction. Secondly, there are no adequate theoretical and methodological approaches that could bring the issue of cognitive development in mathematics into one common theoretical background. It is rather inappropriate to include materials from the Bronze Age and Copper Age[22] using one single research methodology. Whereas for the Bronze Age we can assume the existence of quite complex weight systems[23], we should not expect the same for the Copper Age. However, I also believe, that there is no need to look beyond Europe when searching for the source of mathematical thinking at all. Rather, we should look for an appropriate theoretical construction.

Let us first discuss the analyses presented so far. Can the statistical evidence give credence to the metrological structure of the Cortaillod beads? Yes and no. It is quite a delusion, arising from the approach itself, to assume that there is regularity – an abstract numerical rule in the weight distribution of beads. The appropriate question would be: what lies behind it? Is it that the beads were weighed? Rather not, but all indications so far are that they were valued/measured in some way – meaning that rational methods of measure were applied in their production, as already pointed out by the early studies of the Seeberg deposit[24].

From today's point of view, it is seems odd that four type-weight categories were used by the Cortaillod community. However, the Seeberg find in Switzerland need no longer be considered an oddity, if we put it in a proper historical, economic and technological context. In my earlier work I tried to show that the flint technology related to the production of macrolithic blades was also concerned with dividing and sharing them, usually in four parts[25]. It was the most efficient way of sharing the blades. The relevant parts could reflect specific measurement values such as a half, a quarter, half a quarter, three quarters etc. It cannot have been that much different in the case of the Seeberg beads, which appear like a literal transfer of these manipulations into metal objects. The 'literal' in this case has a double meaning, since it also implies that the determination of certain proportions of the given substance (flint or metal) in the semantic sense was the same or similar. In other words, a vocabulary, which was used for flint and copper, had to be identical for some time.

The different types of beads were perceived not through the abstract measure of weight but through a more specific spatial measure – a linear measure because the Eneolithic communities of Europe were only on the

20 Lefranc *et al.* 2009.
21 Published with the kind permission of Philippe Lefranc.
22 Lenerz De-Wilde 1995; 2002.
23 Petruso 1992; Pare 1999; Peroni 1998.
24 Sangmeister/Strahm 1974.
25 Dzbyński 2008; 2011; 2013.

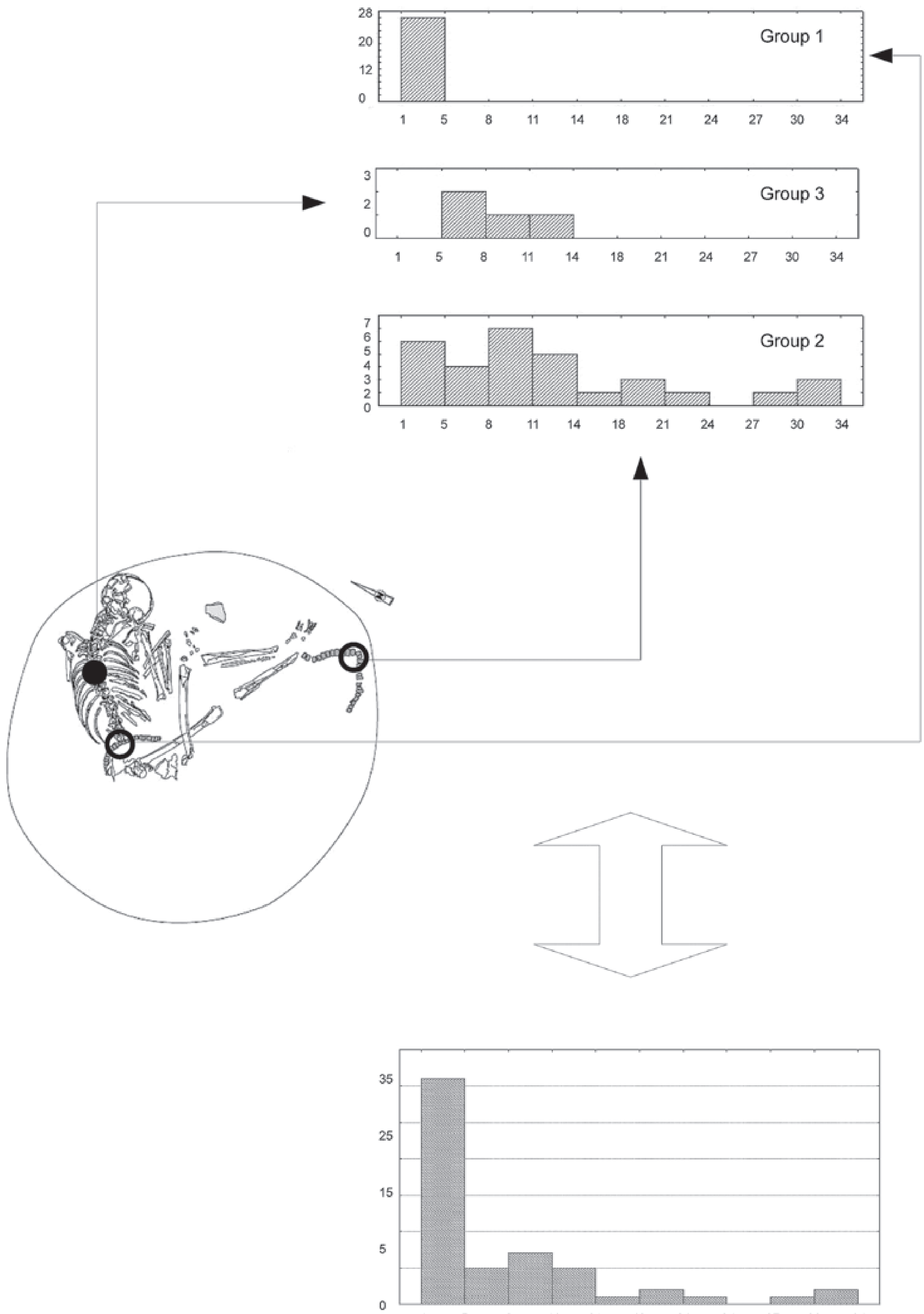

Fig. 5: Position and analysis of the copper beads at Colmar. Certain values (by weight categories) were attached to particular parts of the body

way to a new conceptualisation of metal and close to discovering its wonderful properties. Before the development of more abstract forms of determination by weight measure, which took place probably at the end of the Eneolithic/Bronze Age, metal was perceived rather like stone or flint, as well as treated similarly in its manufacture[26]. A new Eneolithic vocabulary, including the use of abstract measure was not yet ready. This is why four categories of weight-type are observable in the beads of Seeberg. They were linear fragments of metal to which specific number-words were assigned, in a way that is similar to our use today of the terms “one, two, three and four” or “a quarter, a half, one and two” etc.

What could the manufacture of such beads have involved? As the early excavators had already suggested, the beads were made from a copper rod, which was divided into specific segments. The rod underwent plastic forming, resulting in final small bars, which were then knotted to form a bead. Obviously we cannot exclude the possibility that the set of beads at Seeberg was created from several rods at different times. Hypothetically, however, the manufacture of such a set is possible through manipulating and dividing one rod, which I confirmed by dividing once a virtual rod on paper. This can be done for example as shown on Fig. 4. Some fragments of the divided rod had to subsequently be stretched to two or four times their original length to produce an appropriate amount of beads. In other words, the production of such items was a case of the appropriate manipulation of a metal rod and the application of simple mathematical proportion rules.

Theoretical bases

Let us take a look at the idea of the embodied mind and the role of metaphors in human cognition[27]. The ability to create complex metaphors, that is, to mix different areas of meaning (e.g. the area of social relationships with the area of knowledge about the environment or the technological sphere) is, according to Lakoff and Núñez, the fundamental mechanism that allows people to deal with mathematics. This idea is relevant to the conclusions of Mithen[28], who thinks that cognitive fluidity, i.e. the ability to freely combine contents from different fields of human activity – the social sphere, tool making and environment (information on the natural environment) – evolved only among Homo sapiens between 100,000 and 35,000 years ago. The mind of our early ancestors instead operated on a principle of the ‘Swiss-army-knife mentality’, which was much more specialised; therefore combining the technical with the social content was difficult. The symbolisation and metaphorisation of different areas of meaning was almost impossible.

Lakoff and Núñez argue further that the metaphors associated with simple daily activities such as collecting objects in piles, or handling containers containing items, allow us to understand the phenomenon of arithmetic. Abstract mathematical reasoning becomes available to us with a mixture of metaphors (metaphoric blend). This enables us to understand numbers, for example as points plotted on a line. One of the most basic metaphors that allow mathematics to develop is the ‘measuring stick metaphor’.

A measuring stick can be perceived as a physical object, even if it is only an imaginary section of space. But, as Lakoff and Núñez indicate, this object is unidimensional in the sense that it can be infinite. The problem of the infinity of the measuring stick before the invention of philosophy had to be considered at the level of religion and ritual. The abstract version of the measuring stick corresponds to the sections in Euclidean geometry. As a result, this metaphor has the status of a special connection between a physical object and the numbers specifying its size. This metaphor reveals close conceptual analogies to the metaphor of arithmetic as a movement along a line, which allows us to perceive numbers as lying far away or close to each other. It can be added on the margin of our considerations that the consequence of applying the measuring stick metaphor was, in a Euclidean version, the discovery of irrational numbers[29].

I think that in the case of the beads of the Cortaillod culture we are apparently faced with a manifestation of the measuring stick metaphor, i.e. the transformation into objects made of metal. The earlier version of this metaphor was certainly the megalithic yard and its various offshoots proliferating in the archaeological literature, which shall not be discussed here. The transformation of the measuring stick metaphor into objects of metal however had far-reaching consequences, since it introduced the communities using metal to a higher level of abstraction in the perception of numbers and measures. The basic characteristic of copper is its high plasticity that allows to stretch and shorten it, and which is not possible in the case of stone technology. The manipulation of metal had therefore to initiate in the minds of our ancestors questions about

26 Strahm 1994.

27 Lakoff/Johnson 1980; Lakoff/Núñez 2000.

28 Mithen 2003.

29 Lakoff/Núñez 2000, 71.

the nature of measure and number. What was previously invariable became more volatile, and thus it was necessary to invent new ways of conceptualising the new material. We can suspect that it was also realised that the existing concept of the measuring stick bears something more; something invisible that defines the substance without having a form. Developing beads in order to assess their actual 'numerical' values did not make sense, because they were plastically transformed. Their value could be assessed only typologically, as today's archaeologists do. The new material (metal) would soon become defined by its weight, but before it reached this stage among Eneolithic communities, it was used in a way that continued a vocabulary of older provenance. In the case of beads, we can see that the concept of the measuring stick fulfilled its role well.

In the case of the Cortaillod beads we are dealing with tangible evidence of the ongoing discourse on the value of metal in Eneolithic society. And not only this. The beads also shed light on macrolithic industries. They actually are the conceptual glue binding old concepts and vocabularies with the shaping of a new society with metal vocabularies. We see here the development of innovation in this direction, whereas the greater part of manipulation is rooted in the mentality of the Stone Age, based on stone and not metal. The rods and bars, which were formed, incorporate communication processes that were at home among flint industries[30].

Budziszewski[31] points out that the establishment of the macrolithic industry is linked to the development of early metallurgy, as the manufacture and distributional organszation of macroliths was the same as the one used for copper products. After all, many societies which manufactured and used copper were also interested in producing macrolithic tools from good materials. The reason for this might be that, as I have suggested earlier, macroliths were a kind of *alter ego* for metal in a time when both technologies operated within the same complex of words, metaphors and concepts[32]. The basic activity during the distribution and exchange of these idiosyncratic products was their fragmentation, the *signum temporis* of the Eneolithic period which, according to the enchainment idea, was supposed to be a ceremonial means of communication[33]. The properties of metal opened a new layer of underlining innovative forms of fragmentation. It was necessary to adopt more abstract concepts and take on a less concrete way of thinking. We are dealing here, I believe, with a part of the process that was defined by Staaf as a "general common understanding of metallurgy" or even a "new reason" in the cultural discourse of Eneolithic societies[34].

But before that, a different means of assessing the amount of metal was used, which had more in common with the idea of the measuring stick than with an abstract understanding of weight. When we look at the graphs and tables we see imprecise categories, where the weight of beads on the two leather strings overlap (Tab. 1). This could prompt criticism from traditional archaeologists, but it would be a shallow criticism, without understanding the essence of the material; the beads were not yet categorised by weight, although it was probably realised that metal is better defined by something more than its form, such as the length of a metal stick. The full transition to the new system of perception came about in the Bronze Age or slightly earlier when people learned to use abstract weights[35].

Indeed the beads described above are, for our considerations here, a kind of Rosetta Stone. They combine a traditional approach which rests on the more archaic and less abstract methods of the measuring stick with a new approach which uses the plasticity of metal to produce a certain number of beads. It seems that here we can speak of the formation of the concept of portion.

First, the beads lead us to understand that mathematical concepts, or abstract figures, which are well known to us today, were still alien to our Eneolithic ancestors. They belong to a time before certain truths were recognised, which today seem obvious to us. Second, the beads remind us of how these truths were expressed in mathematics. Initially, people learned to judge the length. This was the original concept of the measuring stick, which in my previous work I just called the 'metrological concept'[36]. The idea of the measuring stick was confronted in the Eneolithic with the new metal technology, which allowed so much manipulation that an earlier, specific form could disappear. A longer but thinner stick was still the same stick, i.e. it had the same value since the amount of metal was the same. People had to assess other factors that so far did not have to be considered. It became necessary to take account of the weight of the metal, a new quality. The measuring stick turned into the weight of metal, the amount of length changed into the amount of weight, which formed the basis of true mathematics. Thus numerical abstraction appeared in Europe for the first time.

30 Dzbyński 2008; 2011; 2013.
31 Budziszewski 2006.
32 Dzbyński 2011.
33 Chapman 2000.
34 Staaf 1996.
35 See Pare 1999.
36 Dzbyński 2008.

Concluding remarks

One could question the results presented above on the basis of the post-depositional processes which could have interfered with or even disrupted the actual masses of beads. Indeed in some cases the beads seem to exhibit different oxidation levels, as already mentioned by the initial authors of the Seeberg discovery[37]. Not all weight groups are marked in the same precise way, probably because some of the beads were carried by different people on different strings for a shorter or longer time. We can say that they differ in their 'biographies'. However, as the results presented here clearly show, the observation of the sub-distributions in the histograms is not a problem of today's research. We can therefore assume that, even if post-depositional processes influenced the masses of the beads, their metrological structure remained preserved.

The course of the transformations described in this article, connected with the increasingly rational perception of the value of metal in the Eneolithic period, may be presented as follows. The greater individualisation and internalisation of the measuring stick metaphor in Eneolithic societies, which took place during a period of technical development, resulted in the form of complex fragmentation processes, which should also be understood as a discourse on the value of metal and flint objects[38]. Before the development of the abstract concept of weight, which is the most adequate description of metal in social relations, more specific assessment mechanisms were in use, observable in the fragmentation of copper objects as well as in the shaping of copper into rods according to the measure of length, as is the case of the Cortaillod beads.

The examples and interpretations presented here make us aware of the fact that the process of reaching some truths, which are obvious from our perspective, took place at a time and in a space of which we still know little[39]. We can however assume that mathematics did not spontaneously appear in the heads of our ancestors and that it was not introduced from the outside, but was a long-lasting process, which is active to this day. At this stage we have discussed only part of this process, the very early part. The evidence presented confirms the generally accepted hypothesis that the process of forming mathematical ideas went from the concrete to the abstract. In Europe this was also a process of transforming the measuring stick metaphor into an abstract number concept which belonged to a new vocabulary, describing the metal's weight[40]. According to Renfrew weight is a material-symbolic fact[41]. It does not develop as an embodiment or materialisation of earlier mental concepts but through the development of the concept-construct itself in connection to the experience of the material world. This process took place on a human communication level in interaction with the development of material culture in prehistory.

37 Sangmeister/Strahm 1974.

38 Chapman 2000; Klassen 2001; Staaf 1996.

39 Dzbyński 2013.

40 Dzbyński 2013.

41 Renfrew 2004.

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